

The Initial Mass Function as given by the fragmentation

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Abstract. The dichotomy between a universal mass function (IMF) and a variable IMF which depends on local physical parameters characterises observational and theoretical stellar astronomy. In this contribution the available distributions of probability are briefly reviewed. The physical nature of two of them, gamma variate and lognormal, is then explained once the framework of the fragmentation is introduced. Interpolating techniques are then applied to the sample of the first 10 pc and to the open cluster NGC6649: in both cases lognormal distribution produces the best fit. The three power law function has also been investigated and visual comparison with an artificially generated sample of 100000 stars suggests that the variations in the spectral index are simply due to the small number of stars available in the observational sample. In order to derive the sample of masses, a new formula that allows us to express the mass as a function of the absolute magnitude and (B-V) for MAIN V, GIANTS III and SUPERGIANTS I is derived.

Key words: stars: formation ; stars: statistics ; methods: data analysis ; techniques: photometric

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1. Introduction

The first analytical expression adopted to explain the low mass distribution of stars, see Salpeter (1955), was a power law of the type $\xi(\mathcal{M}) \propto \mathcal{M}^{-\alpha}$ where $\xi(\mathcal{M})$ represents the probability of having a mass between \mathcal{M} and $\mathcal{M} + d\mathcal{M}$; this is called initial mass function, in the following IMF. In the range $10\mathcal{M}_{\odot} > \mathcal{M} \geq 1\mathcal{M}_{\odot}$, the value of α , 2.35, has changed little over the decades and a recent evaluation quotes 2.3, see Kroupa (2001). A fractional exponential law aimed at explaining the mass distribution of asteroids, stars and galaxies was then introduced by Kiang (1966a). A three power law function was later introduced by Scalo (1986), Kroupa et al. (1993), and Binney & Merrifield (1998); in this case the range of existence of the segments as well as the exponent α are functions of the investigated environment. After 15 years, the number of segments was raised to four once a universal IMF was introduced, see Kroupa (2001). Recent developments translate the concept of IMF from our surroundings (for example the first 10 pc) to open clusters such as the Pleiades, see Moraux et al. (2004). A connection between three segment zones and three physical processes has been investigated in Elmegreen (2004); these three zones correspond to brown dwarf masses $\approx 0.02\mathcal{M}_{\odot}$, to intermediate mass stars and high mass stars. The ques-

tion of uniformity or variability of the initial mass function has been carefully explored, see Meusinger (1987), Kroupa (2002), Shadmehri (2004), and Elmegreen (2004).

This paper briefly reviews the physical nature of gamma-variate and lognormal distribution (Section 2.1), introduces an algorithm that deduces the mass from the two basic photometric parameters B-V and M_V (Section 3), and analyses the IMF in the light of physical and non physical functions (Section 4).

2. Preliminaries

Once the histogram of the star's masses is obtained, we can fit it with three different probability density functions (in the following pdf): two of them are well known, the lognormal and the power law, while the third is a gamma-variate which has been applied here for the first time. The lognormal distribution $f_{LN}(x; \mu, \sigma)$ is characterised by the average value μ , and the variance σ^2 , see Evans et al. (2000). The power law distribution has the form $p(x; \alpha) \propto x^{-\alpha}$ where $\alpha > 0$. A refinement of the power law function consists in the multiple-part power law

$$\xi(x) \propto x^{-\alpha_i}, \quad (1)$$

with i varying from 3 to 4. This operation means that the field of existence of the function is broken into i independents zones.

The gamma-variate is analysed in Section 2.1.

2.1. Fragmentation

The stellar mass has been viewed as the Jeans mass in the cloud core, see for example Larson (1992), Elmegreen (1997), Elmegreen (1999), Chabrier (2003), and Bate & Bonnell (2005). Here conversely we adopt the point of view of the fragmentation that was started by Kiang (1966b) where a rigorous demonstration of the size distribution of random Voronoi segments was given. The Kiang pdf (a gamma-variate) has the form

$$H(x; c) = \frac{c}{\Gamma(c)} (cx)^{c-1} e^{-cx}, \quad (2)$$

where $0 \leq x < \infty$, $c > 0$, and $\Gamma(c)$ is the gamma function with argument c , see Kiang (1966b). This pdf is characterised by average value, $\mu=1$, and variance, $\sigma^2=1/c$.

Starting from a rigorous result in 1D, Kiang (1966b) conjectured that the Voronoi cells in 2D/3D have an area/volume distribution represented by equation (2) with $c=4/6$.

We now briefly review a useful result that can be obtained once $x_1 \cdots x_N$, independent random variables, are introduced. Supposing that

$$y = x_1 \times x_2 \times \cdots \times x_{N-1} \times x_N = \prod_{j=1}^N x_j, \quad (3)$$

it can be shown that for large N the pdf of y is approximately lognormal, see Hwei Hsu (1996). From this point of view the area with $y = x_1 \times x_2$ or the volumes $y = x_1 \times x_2 \times x_3$ can be fitted with a lognormal once the number 2 and 3 are considered a transition to large N .

3. Mass determination from colour absolute-magnitude

The (B-V) colour can be expressed as

$$(B - V) = K_{BV} + \frac{T_{BV}}{T}, \quad (4)$$

here T is the temperature, K_{BV} and T_{BV} are two parameters that can be derived by implementing the least square method on a series of calibrated data. The bolometric correction, BC , can be expressed as

$$BC = M_{bol} - M_V = -\frac{T_{BC}}{T} - 10 \log_{10} T + K_{BC}, \quad (5)$$

where M_{bol} is the absolute bolometric magnitude, M_V is the absolute visual magnitude, T_{BC} and K_{BC} are two parameters that can be derived through the general linear least square method applied to a series of calibrated data. We now justify the theoretical basis of equations (4) and (5). We can start from the definition of (B-V)

$$(B - V) = m_B - m_V = K - 2.5 \log_{10} \frac{\int S_B I_\lambda d\lambda}{\int S_V I_\lambda d\lambda}, \quad (6)$$

where S_λ is the sensitivity function in the region specified by the index λ , K is a constant and I_λ is the energy flux reaching

the earth. We now define a sensitivity function for a pseudo-monochromatic BV system

$$S_\lambda = \delta(\lambda - \lambda_i) \quad i = B, V, \quad (7)$$

where δ denotes the Dirac delta function. On inserting the energy flux as given by the Planck distribution, the following is found

$$(B - V) = K' - \frac{hc}{kT} \left(\frac{\lambda_B - \lambda_V}{\lambda_B \lambda_V} \right) 2.5 \log_{10}(e), \quad (8)$$

where K' is a constant, c is the velocity of light, k is the Boltzmann's constant, h is the Planck's constant, λ_B the wavelength of the filter B, λ_V the wavelength of the filter V, and T the temperature. On inserting $\lambda_B = 4450 \text{ \AA}$ and $\lambda_V = 5500 \text{ \AA}$ in equation (8), the equation (4) is theoretically derived.

A theoretical couple, $T_{BV}=6701 \text{ K}$ and $K_{BV}=-0.51$ can be found by inserting the constants of the Planck distribution in equation (8) and calibrating the relationship on the Sun couple, $T_\odot=5777 \text{ K}$ and $(B - V)_\odot=0.65$. In a similar way the bolometric correction is found to be

$$BC = M_{bol} - M_V = K'' - 2.5 \log_{10} \frac{\int_0^\infty I_\lambda d\lambda}{\int S_V I_\lambda d\lambda}, \quad (9)$$

where K'' is another constant.

When the numerical value of the exponential in the Planck function is much greater than one, equation (5) is found with the theoretical value $T_{BC}=28402 \text{ K}$ and $K_{BC}=42.45$, once BC of the sun is requested, $BC_\odot=-0.08$ at $T_\odot=5777 \text{ K}$.

The application of equation (4) and (5) to the calibrated physical parameters for stars of the various luminosity classes has been reported in Table 1.

Table 1. Table of coefficients from calibrated data.

	MAIN, V	GIANTS, III,	SUPERGIANTS I
K_{BV}	-0.641 ± 0.01	-0.792 ± 0.06	-0.749 ± 0.01
$T_{BV}[\text{K}]$	7360 ± 66	8527 ± 257	8261 ± 67
K_{BC}	42.74 ± 0.01	44.11 ± 0.06	42.87 ± 0.01
$T_{BC}[\text{K}]$	31556 ± 66	36856 ± 257	31573 ± 67
a_{LM}	0.062 ± 0.04	0.32 ± 0.14	1.29 ± 0.32
b_{LM}	3.43 ± 0.06	2.79 ± 0.23	2.43 ± 0.26

These numerical results can be visualised in Figure 1 when (B-V) against T is considered and in Figure 2 where the relationship between BC and T is reported; see Press et al. (1992) about the fitting techniques.

Other authors analyse the form $T=T((B-V), [\text{Fe}/\text{H}], \log_{10} g)$ where $[\text{Fe}/\text{H}]$ represents the metallicity and g the surface gravity, see Sekiguchi & Fukugita (2000), or a two piecewise function for $B-V=f(T)$, see for example Reed (1998).

The luminosity of the star is represented through the usual formula

$$\log_{10} \left(\frac{L}{L_\odot} \right) = 0.4(4.74 - M_{bol}), \quad (10)$$

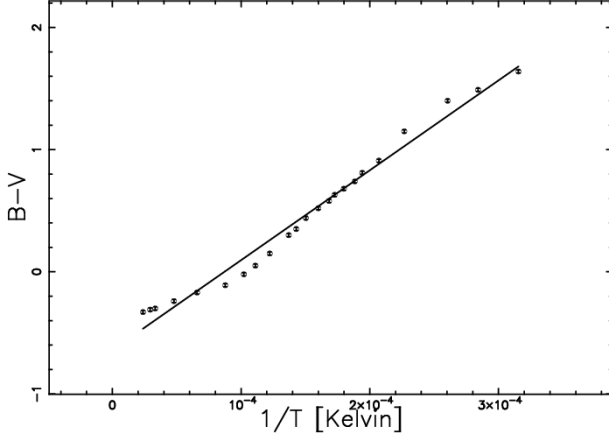


Fig. 1. Least square fit of (B-V) against $1/T$, MAIN SEQUENCE V. Error on (B-V)=0.025. The calibrated data, extracted from Table 15.7 in Drilling & Landolt (2000), are represented with their error, and the full line reports the suggested fit.

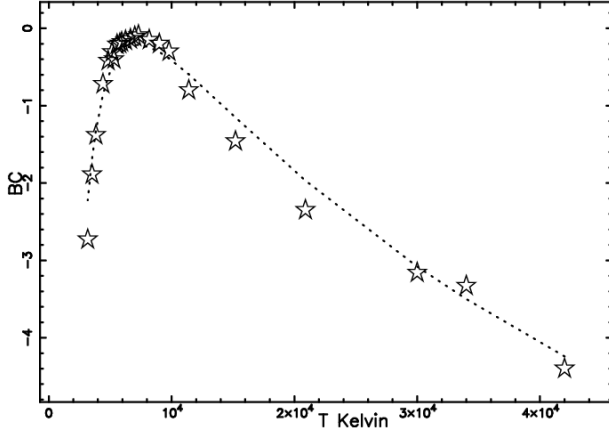


Fig. 2. Relationship of BC against T , MAIN SEQUENCE V. Error on BC=0.025. The stars represent the calibrated data extracted from Table 15.7 in Drilling & Landolt (2000) and the dotted line, the suggested fit.

see, for example, equation (5.120) in Lang (1999). The connection with the mass is made through the numerical relationship

$$\log_{10}\left(\frac{L}{L_{\odot}}\right) = a_{LM} + b_{LM} \log_{10}\left(\frac{\mathcal{M}}{\mathcal{M}_{\odot}}\right) \quad (11)$$

for $\mathcal{M} > 0.2\mathcal{M}_{\odot}$.

The numerical coefficients a_{LM} and b_{LM} , as deduced from the calibrated data in Table 3.1 of Deeming & Bowers (1984) are reported in Table 1.

With the theory here adopted the mass of the star is

$$\log_{10} \frac{\mathcal{M}}{\mathcal{M}_{\odot}} = \frac{-0.4 M_V - 0.4 K_{BC} + 4.0 \ln \left(\frac{T_{BV}}{(B-V) - K_{BV}} \right) \left(\ln(10)^{-1} \right)}{b_{LM}} - \frac{0.4 \frac{T_{BC}(-(B-V) + K_{BV})}{T_{BV}} + 1.896 - a_{LM}}{b_{LM}} \quad (12)$$

From a practical point of view the percentage of reliability of our evaluation can also be introduced:

$$\epsilon = \left(1 - \frac{|\mathcal{M}^{\text{cal}} - \mathcal{M}^{\text{num}}|}{\mathcal{M}^{\text{cal}} + \mathcal{M}^{\text{num}}} \right) \times 100, \quad (13)$$

where \mathcal{M}^{cal} is the calibrated value of the star mass, see Table 15.8 in Drilling & Landolt (2000), and \mathcal{M}^{num} is the numerical value here evaluated. The resulting masses are reported in Table 2.

Table 2. Table of evaluated masses through an analytical method, equation (12).

sp	MK Class	$\mathcal{M}^{\text{cal}}/\mathcal{M}_{\odot}$	$\mathcal{M}^{\text{num}}/\mathcal{M}_{\odot}$	$\epsilon(\%)$
B0	V	17.5	17.34	99.54
A0	V	2.9	3.42	91.7
F0	V	1.6	1.76	95.23
G0	V	1.05	1.13	96.02
K0	V	0.79	0.81	98.73
M2	V	0.4	0.38	98.38
K5	III	1.2	6.38	31.63
M0	III	1.2	7.15	28.71
A0	I	16.0	22.35	83.44
M0	I	13.0	27.73	63.82
M2	I	19.0	28.6	79.82

The formulae here derived can be used to deduce the mass of the stars once the parameters M_V , B-V and luminosity class are provided.

4. Masses

We now apply the developed techniques to the nearest stars and to an open cluster, NGC6649.

4.1. The masses in the first 10 pc

The completeness in masses of the sample can be evaluated in the following way. The limiting apparent magnitude m_v^L is known, for example in the case of Hipparcos it is 8. The corresponding absolute limiting magnitude is computed and inserted in equation (12). The limiting mass, \mathcal{M}^L , for stars belonging to the MAIN SEQUENCE V is

$$\log_{10} \frac{\mathcal{M}^L}{\mathcal{M}_{\odot}} = -0.1164 m_v^L + 0.2528 \ln(0.1 d_1) - 4.122 + 0.505 \ln \left(\frac{0.368 10^{11}}{0.5 10^7 (B-V) + 0.3206 10^7} \right) + 0.499 (B-V) \quad (14)$$

where d_1 is the maximum distance that characterises the sample in pc. When $d_1=10$, $m_v^L=8$, and (B-V)=1.15 we obtain $\mathcal{M}^L = 0.53\mathcal{M}_{\odot}$; this is the limit over which the sample in masses is complete.

Due to the fact that Hipparcos (ESA 1997) gives B-V, m_V and parallax, we can implement our algorithm on that database. The classification in MK classes (I,III,V) is then obtained by a comparison with the interval of existence of

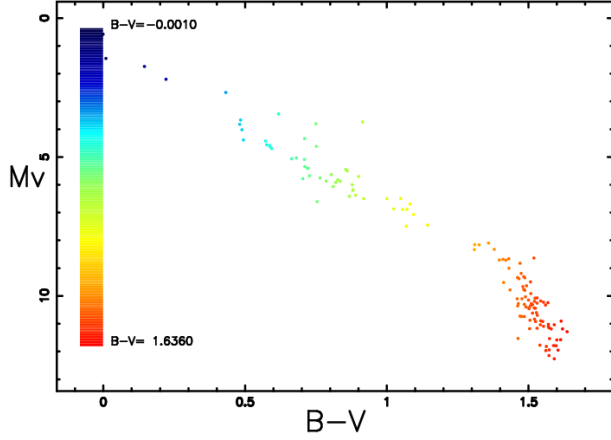


Fig. 3. M_V against $(B-V)$ (H-R diagram) in the first 10 pc, this sample is incomplete in masses.

calibrated stars. In Figure 3 we report the H-R diagram of the first 10 pc when the incomplete (in masses) sample is considered.

In Table 3 we report the coefficients of the three pdf, average value, the number, N_s , of stars and the merit function χ^2 . A typical fit of the complete sample in the light of the power law pdf is reported in Figure 4, conversely Figure 5 reports the fit through the lognormal distribution.

Table 3. Coefficients of mass distribution in the first 10 pc, of the incomplete sample, GIANTS included, $\mathcal{M} \geq 0.2\mathcal{M}_\odot$, $\overline{\mathcal{M}} = 0.61\mathcal{M}_\odot$ and $N_s=137$.

Kiang	$\chi^2=14.28$	$c=1.67$	
Power law	$\chi^2=17.6$	$\alpha=1.85$	
lognormal	$\chi^2=0.95$	$\mu=0.6$	$\sigma=0.57$

Table 4. Coefficients of mass distribution in the first 10 pc, of the complete sample, $\mathcal{M} \geq 0.53\mathcal{M}_\odot$, $\overline{\mathcal{M}} = 0.98\mathcal{M}_\odot$, GIANTS excluded, and $N_s=57$.

Kiang	$\chi^2=12.62$	$c=3.42$	
Power law	$\chi^2=8.7$	$\alpha=2.42$	
lognormal	$\chi^2=4.55$	$\mu=1.01$	$\sigma=0.41$

4.2. The masses in open clusters

The distribution of masses in open clusters can be considered a further application of the pdf here studied. We briefly review some of the typical clusters subject to investigation: IC348 (Preibisch et al. 2003 ; Muench et al. 2003 ; Luhman et al. 2003), Taurus (Luhman 2000 ; Briceño et al. 2002), Orion trapezium (Hillenbrand & Carpenter 2000 ; Luhman et al. 2000 ; Muench et al. 2002), Pleiades (Bouvier et al. 1998 ; Luhman et al. 2000), and M35 (Barrado y Navascués et al. 2001).

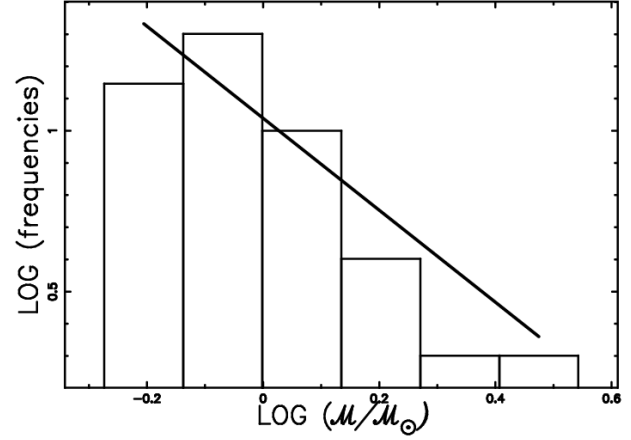


Fig. 4. Log-Log histogram of mass distribution with a superposition of the fitting line (the power law distribution) when the complete sample is considered. Parameters as in Table 3.

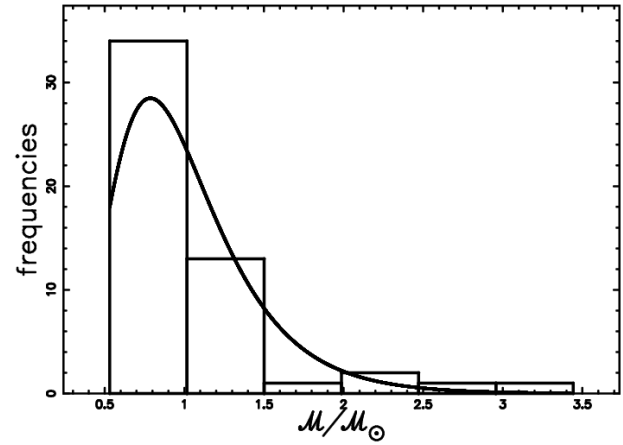


Fig. 5. Histogram (step-diagram) of mass distribution with a superposition of the lognormal distribution when the complete sample is considered. Parameters as in Table 3.

Here attention is focused on NGC6649, see Walker & Laney (1987). In this case the distance modulus is 11 (1585 pc) and this fact allows the determination of the masses from M_V – $(B-V)$ diagram, see Figure 6, once the algorithm outlined in Sect. 3 is adopted.

In Table 5 we report the parameters relative to the three pdf. Once the framework of the three power law is accepted, Table 6 reports the results that are visualised in Figure 7.

Table 5. Coefficients of mass distribution in NGC 6649 where $N_s=234$ and $\overline{\mathcal{M}} = 1.04\mathcal{M}_\odot$.

Kiang	$\chi^2=35.44$	$c=5.02$	
Power law	$\chi^2=248.49$	$\alpha=3.12$	
lognormal	$\chi^2=29.39$	$\mu=1.03$	$\sigma=0.32$

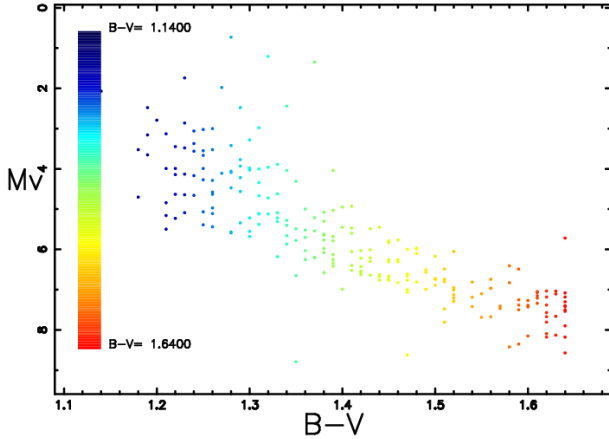


Fig. 6. M_V against (B-V) (H-R diagram) in NGC6649.

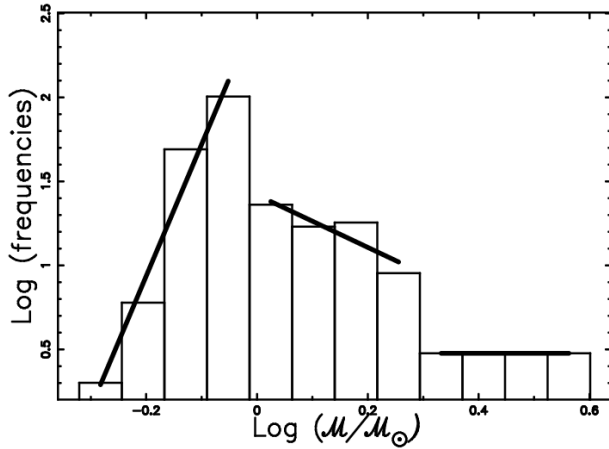


Fig. 7. Log-Log histogram of mass distribution in NGC6649 with a superposition of the three-part power law. Parameters as in Table 6.

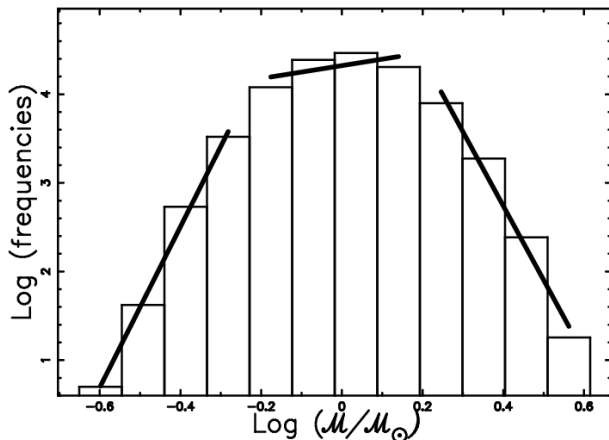


Fig. 8. Log-Log histogram of 100000 masses randomly generated according to lognormal distribution (parameters as in Table 6) with a superposition of the three-part power law.

Table 6. Three power law in NGC 6649 where $N_s=234$.

Interval	α
$0.52 \leq M/M_\odot < 0.88$	-6.83
$0.88 \leq M/M_\odot < 1.8$	2.55
$1.80 \leq M/M_\odot < 3.65$	1.0

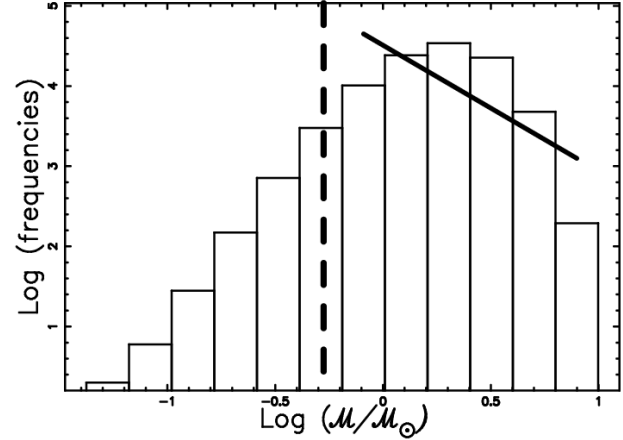


Fig. 9. Log-Log histogram of 100000 masses randomly generated according to the Kiang distribution with $c=4$ and $\bar{M} = 2M_\odot$. On the right, $M \geq 0.51M_\odot$, is visible the fitting power law distribution with $\alpha=2.56$. The vertical dashed line represents the limiting mass $M^L = 0.53M_\odot$.

5. Conclusions

A new method represented by equation (12), allows us to deduce the mass of stars from photometric data. This method is limited in mass, see equation (14), in the same way as the Hipparcos data are complete up to $m_v=8$.

In the second part of this paper the obtained mass distribution is fitted with the pdf of physical nature: the lognormal and the gamma variate. The gamma-variate is applied for the first time as a fitting function of mass distribution. When the complete sample of masses is fitted with the gamma-variate, $c=3.42$, is found; a comparison should be made with $c=4$, which is the value suggested by Kiang (1966b) in the case of 2D fragmentation. Careful analysis of the results of Table 5 and Table 4 indicates that the lognormal gives the best fit to the star's masses. On the contrary the three power law function simply represents the various segments of a continuous curve. This point of view can be expressed by plotting 100000 stars generated according to lognormal data, see Figure 8.

In Figure 8, the three indices α_i are now varying in a continuous way. The great variations in α_i visible in Table 6 and Figure 7, can be due to the transition from a great number of stars (smooth behaviour of α_i) to a small number of stars (jumps in α_i or variability in the IMF).

It is now possible in the light of the limiting mass M^L , see equation (14), to explain the reason for which a power law fits the masses well in the first 10 pc. The limiting mass introduces a cut in the physical sample at a value that is probably near the averaged value, and therefore only the masses of value greater than the averaged value are detected. This point of view is simulated in Figure 9 in which a real con-

tinuous mass distribution as given by a gamma-variate with $c=4$ is reported: due to the limiting mass \mathcal{M}^L a power law distribution is observed.

Sekiguchi, M., & Fukugita, M. 2000, *AJ* , 120, 1072
 Shadmehri, M. 2004, *MNRAS* , 354, 375
 Walker, A. R., & Laney, C. D. 1987, *MNRAS* , 224, 61

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References

Barrado y Navascués, D., Stauffer, J. R., Bouvier, J., & Martín, E. L. 2001, *ApJ* , 546, 1006
 Bate, M. R., & Bonnell, I. A. 2005, *MNRAS* , 356, 1201
 Binney, J. & Merrifield M. : 1998, *Galactic Astronomy*, Princeton University Press, Princeton, p. 286
 Bouvier, J., Stauffer, J. R., Martin, E. L., Barrado y Navascués, D., Wallace, B., & Bejar, V. J. S. 1998, *A&A*, 336, 490
 Briceño, C., Luhman, K. L., Hartmann, L., Stauffer, J. R., & Kirkpatrick, J. D. 2002, *ApJ* , 580, 317
 Chabrier, G. 2003, *PASP* , 115, 763
 Drilling, J. S. & Landolt, A. U. in N. Cox (editor): 2000, *Allen's Astrophysical Quantities*, Springer, Berlin, pp. 381-396
 Deeming, T., & Bowers, R. L.: 1984, *Astrophysics II: Interstellar Matter and Galaxies*, Jones & Bartlett Pub, Boston
 Elmegreen, B. G. 1997, *ApJ* , 486, 944
 Elmegreen, B. G. 1999, *ApJ* , 515, 323
 Elmegreen, B. G. 2004, *MNRAS* , 354, 367
 ESA : 1997, *The Hipparcos Catalogue (ESA SP-1200)*, ESA, Noordwijk
 Evans M., Hastings N., Peacock B. : 2000, *Statistical Distributions - third edition*, John Wiley & Sons Inc., New York
 Hwei P. Hsu : 1996, *Probability, Random variables, & Random Processes* McGraw-Hill, New York
 Hillenbrand, L. A., & Carpenter, J. M. 2000, *ApJ* , 540, 236
 Kiang, T. 1966a, *Z. Astrophys.*, 64, 426
 Kiang, T. 1966b, *Z. Astrophys.*, 64, 433
 Kroupa, P. 2001, *MNRAS* , 322, 231
 Kroupa, P. 2002, *Science*, 295, 82
 Kroupa, P., Tout, C. A., & Gilmore, G. 1993, *MNRAS* , 262, 545
 Lang, K. R.: 1999 *Astrophysical Formulae*, third edition, Springer, Berlin
 Larson, R. B. 1992, *MNRAS* , 256, 641
 Luhman, K. L. 2000, *ApJ* , 544, 1044
 Luhman, K. L., Rieke, G. H., Young, E. T., Cotera, A. S., Chen, H., Rieke, M. J., Schneider, G., & Thompson, R. I. 2000, *ApJ* , 540, 1016
 Luhman, K. L., Stauffer, J. R., Muench, A. A., Rieke, G. H., Lada, E. A., Bouvier, J., & Lada, C. J. 2003, *ApJ* , 593, 1093
 Meusinger, H. 1987, *Astronomische Nachrichten*, 308, 189
 Moraux, E., Kroupa, P., & Bouvier, J. 2004, *A&A*, 426, 75
 Muench, A. A., Lada, E. A., Lada, C. J., & Alves, J. 2002, *ApJ* , 573, 366
 Muench, A. A., et al. 2003, *AJ* , 125, 2029
 Preibisch, T., Stanke, T., & Zinnecker, H. 2003, *A&A*, 409, 147
 Press, W. H. , Teukolsky, S. A. , Vetterling, W. T. , & Flannery, B. P.: 1992 *Numerical Recipes in Fortran*, Cambridge University Press, Cambridge
 Reed, B. C. 1998, *JRASC*, 92, 36
 Salpeter, E. E. 1955, *ApJ* , 121, 161
 Scalo, J. M. 1986, *Fundamentals of Cosmic Physics*, 11, 1